FOUR 4S

"Mathematics is, in its way, the poetry of logical ideas."

—Albert Einstein

OVERVIEW

Students work together to figure out different ways of achieving the same answer through the writing of equivalent expressions.

PLANNING FOR IT

WHEN YOU MIGHT USE THIS PRACTICE

• To build a growth mindset culture for students around mathematics class
• To promote the value of creativity and different strategies in math
• To create a productive climate of equitable group work in mathematics class
• In the early days of the school year to help students get used to going to the board

TIME REQUIRED

• ≤ 30 minutes, at one time or returned to over a few days

LEVEL

• Upper Elementary
• Middle School
• High School

MATERIALS

• Blank journals (optional)
LEARNING OBJECTIVE

Students will:

- Develop a growth mindset toward mathematics
- Learn to participate in different ways in mathematics class
- Become more comfortable sharing their work on the board
- Help each other with mathematical thinking

ADDITIONAL SUPPORTS

- Making Practices Culturally Responsive
- Adapting Practices for Students with Special Needs
- Making Classrooms and Schools Trauma-Informed and Healing-Centered

SEL COMPETENCIES

- Social Awareness
- Relationship Skills

HOW TO DO IT

REFLECTION BEFORE THE PRACTICE

- Take a moment to reflect on how you felt about math when you were your students’ age. How do you want your students to feel about math?
- What can you do to uncover and deal with any biases you might have with regards to the students in your math class? What steps can you take to ensure equitable participation from all students?

INSTRUCTIONS

Note: Practice 4 of 5 in youcubed Mathematical Mindset practice collection.

TO THE TEACHER

Four 4s encourages all students to participate in different ways and allows them to get comfortable sharing their work and helping each other. It’s a great way to start the year because it is an opportunity to model what to do with mistakes when we see them, how mistakes are good to share, and how there are no bad consequences for making them. It also encourages students to look for more than one way to solve a problem.

Another message this activity communicates is that some problems take time and it’s okay if we don’t complete an activity. We can work on an activity and go back to it over time. When we go back to it we can be more creative and find other expressions. Four 4’s pushes students to go beyond the practice of just finding an answer and has them working to find many different ways to achieve and answer.
INTRODUCTION

This opening activity provides students the opportunity to appreciate different ways of achieving the same answer through the writing of equivalent expressions. During this activity students can practice orders of operation and work together to find different expressions that result in the same numerical answer. Mistakes are valued and creative ways of making an answer are celebrated. This activity also helps students get comfortable coming to the board -- in a safe, low risk way.

LAUNCH

• Start the activity by writing the numbers 1 to 20 on the board with plenty of space in between them. Ask students to record these numbers in their journal as well.

• If students are familiar with four 4’s, you can replace it with another activity like three 2’s and two 3’s or 1·4·9·2.

• Tell students in this activity they are trying to find every number from 1 to 20 using only four 4’s – all four of them have to be used each time – and any operations. Ask them to think of operations and make a list on the board of what they come up with.

• Let students know you want them to include as many strategies as they can think of for each number and to write this in their journals and on the board.
  • Tell them that if they come up with something for a number that already has a strategy, you want them to use a different color and write their strategy on the board too.
  • Remind them that there are many ways to approach these problems and that is one of the things you want them to experience when working on this activity.

• Let them know that while the finding of individual strategies is not a very group-worthy task--and students often like to work alone in thinking about them--you have them sitting in groups so they can discuss their ideas and strategies.

EXPLORE (5 MINUTES)

• While students are exploring, encourage them to put their strategies on the board whenever they think of one. Remind them they can put up as many solutions as they can think of for each number.
  • If you notice a student puts up an expression with a mistake, do not identify it as incorrect; wait, as students will often see it for themselves and correct it as more solutions are shared.
  • Make space for students to explore the activity at their own pace. This is a useful activity to come back to over a few days, so there is no need to rush them along. When you come back to the activity, have students work on the numbers that they have not yet found solutions for.
DISCUSS (10+ MINUTES)

- Students will likely find all of the numbers except 11, 13, and 19. At that point, you can share with students the meaning of the factorial operation (if appropriate).
  - We recommend introducing it by just writing the equivalent expressions for 1 through 4 factorial on the board. Let students read it and work to understand the patterns in what you have written. It is important to do this so students become used to reading for understanding, noticing patterns and repeated steps in reasoning.
  - If students do need to discuss how factorial works, they can do that in their groups. Students will put factorial to good use, as they will see that it allows them to find the missing numbers and new ways to make numbers they have already found.

EXTEND

- Can you continue using Four 4’s to find numbers greater than 20?
- Make a number challenge of your own that is similar to Four 4’s.
- Can you use Four 4’s to make negative numbers?
- How many numbers can you make with Five 5’s?

LOOK-FORS

- **What numbers are students finding?** What solutions are students coming up with? Are they finding more than one solution? If they are finding different solutions for one number invite the students to put those different solutions up on the board.
- **Are students using different operations?** Is there an operation they aren’t using? Do students understand that they can use parentheses? Watch to see if any students use new operations that the class didn’t think of. If a student is using exponents or factorials they could share those with the class. Many students get excited about this problem and will enjoy thinking of other solutions on their own. It is also a great task for students to pull out if they have finished another task. Keep it up in the classroom so that as a class you can develop your solutions over time.
- **Which students are putting solutions up on the board?** Look for the solutions students are finding for each number. Encourage students to put their solutions up on the board. If there are students who haven’t been up to the board during the week, try to pay attention to the solutions they are finding. This is a great way for students to put their work up for the first time.
- **Notice if students are putting up solutions that don't work. Are other students noticing?** If a student notices something doesn’t work have them adjust the solution in a different color. You can pull together the class at some point and have conversations about these different solutions and why some do or do not work.
- **How are students communicating solutions?** Invite students to share their thinking and their findings with their group when they have had some time to think individually. Something one student says may inspire another student with a new idea. This may also spark some conversations about order of operations and how moving numbers and operations around can change the solution.
REFLECT (5 MINUTES)

- Ask students to respond to the reflection questions in their math journals: What surprised you about this activity? What did you learn from this activity?

REFLECTION AFTER THE PRACTICE

- How did students respond when/if they or others made mistakes?
- Did this activity help students to see math as enjoyable and collaborative? To what extent did it provide a way for students to work together as equitable members of a group?
- Can you think of other ways of bringing this style of creatively working together into your math class?

THE RESEARCH BEHIND THE PRACTICE

EVIDENCE THAT IT WORKS

Research has shown that students who learned about growth mindset with regards to mathematics reported more positive beliefs about math, were more engaged in math class, and did better on standardized math achievement tests. Mindset interventions in math benefit all students, but have demonstrated even more power for groups that may be more affected by myths about math learning, including girls, English language learners, and economically disadvantaged students.

In addition, a four-year study of high school students in different types of math classes showed that the students who learned math in mixed-ability classrooms that emphasized cooperative group work, open problem-solving, and the use of multiple strategies--compared to those in traditional math classrooms, which were often ability-grouped and focused on teacher lectures and individual work--demonstrated greater gains in math achievement and greater reductions in achievement gaps, enjoyed math more, and treated each other with more respect, support, and equity.

WHY DOES IT MATTER?

A substantial body of research has indicated that students who have a growth mindset about intelligence--who believe that, with effort, intelligence can be changed over time--are more likely to do well academically.

Importantly, evidence shows that growth mindset can be learned: in a nationally representative study, students who were taught about a growth mindset of intelligence went on to earn better grades (especially if they started out lower-achieving) and select more challenging classes. Grades improved even more in schools with more supportive learning climates, in which peer norms supported the growth mindset message.

Though much of the research on growth mindset has to do with beliefs about intelligence, other research suggests that social and emotional growth mindsets (e.g., believing that personality, emotions, etc., can grow and change) can reduce bias and promote well-being, social competence, and prosocial behavior.
This is a practice from youcubed, a center at Stanford University led by Professor Jo Boaler. In addition to classroom ideas and videos, youcubed offers a variety of resources for mathematics educators, including research summaries and professional development.